

Introduction

PFAS (“forever chemicals”) resist conventional water treatment because the carbon–fluorine bond is exceptionally strong. Ultrasound-driven argon nanobubbles offer a route to destroying them: PFAS are surface-active, so they adsorb at the bubble–water interface, and the violent collapse of that bubble under ultrasound generates temperatures high enough to cleave C–F bonds [3]. **Collapse severity is therefore the key lever for destruction efficiency.**

A sonoreactor holds a *cloud* of bubbles, not one bubble. The ultrasound-driven dynamics of the bubbles are fully-coupled: neighbors may attract or repel, and influence collapse and peak temperature [1].

Predicting destruction efficiency is therefore a coupled multi-bubble problem. We apply an extension of the Rayleigh–Plesset (RP) framework [1] to a cloud of N nanobubbles coupled through the surrounding acoustic field, in which each bubble’s dynamics depends on pressure radiated by its neighbors. For the single-bubble limit, see the companion poster; see also [2].

Governing Equations

Lengths are scaled by the reference bubble radius R_{ref} , time by ω^{-1} (so $\tau = \omega t$, where $\omega = 2\pi f$ is the ultrasound angular frequency), and pressures by the ambient pressure P_0 ; P_g is the gas pressure inside the bubble, $P_a(t) = -p_A \sin(\omega t)$ the applied acoustic pressure, ρ the liquid density, μ the viscosity, σ the surface tension, and c the speed of sound. The nondimensional radius $r_n(\tau)$ of the n -th bubble ($n = 1, \dots, N$) obeys an extended RP equation [1], with $\dot{} = d/d\tau$:

$$\beta \left(r_n \ddot{r}_n + \frac{3}{2} \dot{r}_n^2 \right) = \hat{P}_g - \hat{P}_a - 1 + \chi r_n \frac{d}{d\tau} (\hat{P}_g - \hat{P}_a) - \frac{\eta \dot{r}_n}{r_n} - \frac{\hat{\sigma}}{r_n} - \beta \sum_{l \neq n} \frac{r_l^2 \ddot{r}_l + 2r_l \dot{r}_l^2}{d_{nl}} \quad (\text{pairwise interaction})$$

where the nondimensional applied acoustic pressure is $\hat{P}_a(\tau) = -\hat{p}_A \sin(\tau)$, and the dimensionless groups are $\beta = \rho R_{\text{ref}}^2 \omega^2 / P_0$, $\chi = \omega R_{\text{ref}} / c$, $\eta = 4\mu\omega / P_0$, $\hat{\sigma} = 2\sigma / (P_0 R_{\text{ref}})$. The nondimensional position $\mathbf{x}_n(\tau)$ of each bubble center then obeys a coupled translational equation [1]:

$$\ddot{\mathbf{x}}_n = \hat{\mathbf{F}}_n - \frac{3\dot{r}_n \dot{\mathbf{x}}_n}{r_n} \quad (\text{self radial-translational}) + \sum_{l \neq n} \left\{ \frac{3(\mathbf{x}_n - \mathbf{x}_l)}{d_{nl}^3} \left(r_l^2 \ddot{r}_l + 2r_l \dot{r}_l^2 + \frac{\dot{r}_n \dot{r}_l r_l^2}{r_n} \right) \right. \\ (\text{secondary Bjerknes — pressure radiated by neighbor } l) \\ \left. - \frac{3r_l^2}{2d_{nl}^3} \left[r_l \ddot{\mathbf{x}}_l + \left(\frac{\dot{r}_n}{r_n} r_l + 5\dot{r}_l \right) \dot{\mathbf{x}}_l \right] + \frac{9r_l^2(\mathbf{x}_n - \mathbf{x}_l)}{2d_{nl}^5} \left[(\mathbf{x}_n - \mathbf{x}_l) \cdot \left(r_l \ddot{\mathbf{x}}_l + \left(\frac{\dot{r}_n}{r_n} r_l + 5\dot{r}_l \right) \dot{\mathbf{x}}_l \right) \right] \right\}$$

Governing Equations (cont.)

where $\hat{\mathbf{F}}_n = \hat{\mathbf{F}}_{B1,n} + \hat{\mathbf{F}}_{d,n}$, with

$$\hat{\mathbf{F}}_{B1,n} = -\frac{2\chi \hat{p}_A}{\beta} \sin(\chi x_n) \sin(\tau) \hat{\mathbf{i}} \\ \hat{\mathbf{F}}_{d,n} = -\frac{9\eta}{2\beta r_n^2} \left(\dot{\mathbf{x}}_n - \mathbf{v}_{ac} - \sum_{l \neq n} \mathbf{v}_{ln} \right)$$

the **primary Bjerknes** force (pressure gradient, along wave direction $\hat{\mathbf{i}}$) and the **Levich drag** (relative motion against $\mathbf{v}_{ac}$ and neighbor-induced flow $\mathbf{v}_{ln}$). Term colors match the corresponding force curves in the plots below.

Results

We present results for 2- and 3-bubble systems, examining how bubble interactions depend on acoustic forcing amplitude.

Two equal-size bubbles, $\hat{p}_A = 2$

$r_{0,1} = r_{0,2} = 1$, 4 acoustic cycles, coupled vs. uncoupled.

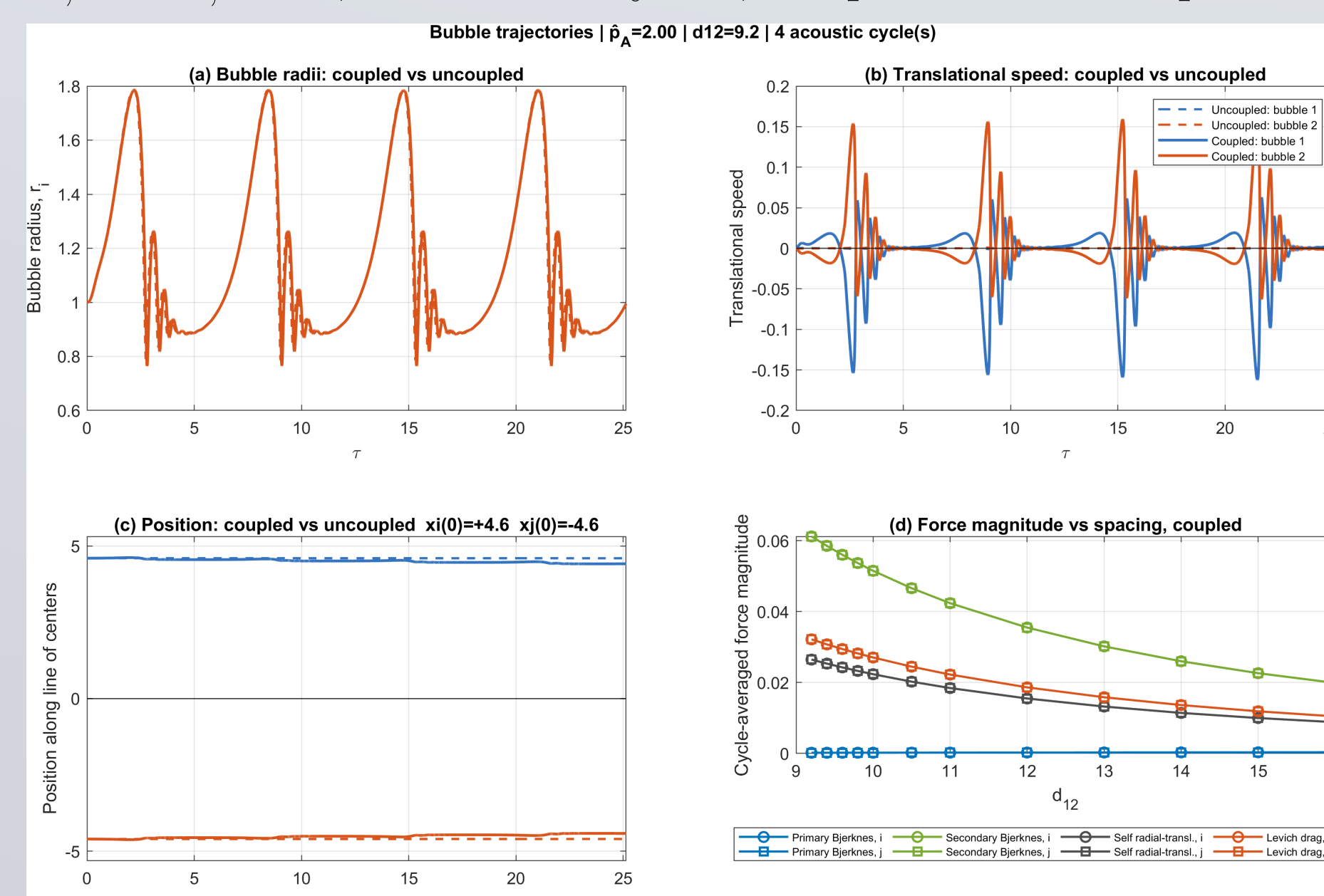


Figure 1: Radius, speed, and position vs. time; force vs. spacing.

Two equal-size bubbles, $\hat{p}_A = 3$

$r_{0,1} = r_{0,2} = 1$, 4 acoustic cycles, coupled vs. uncoupled.

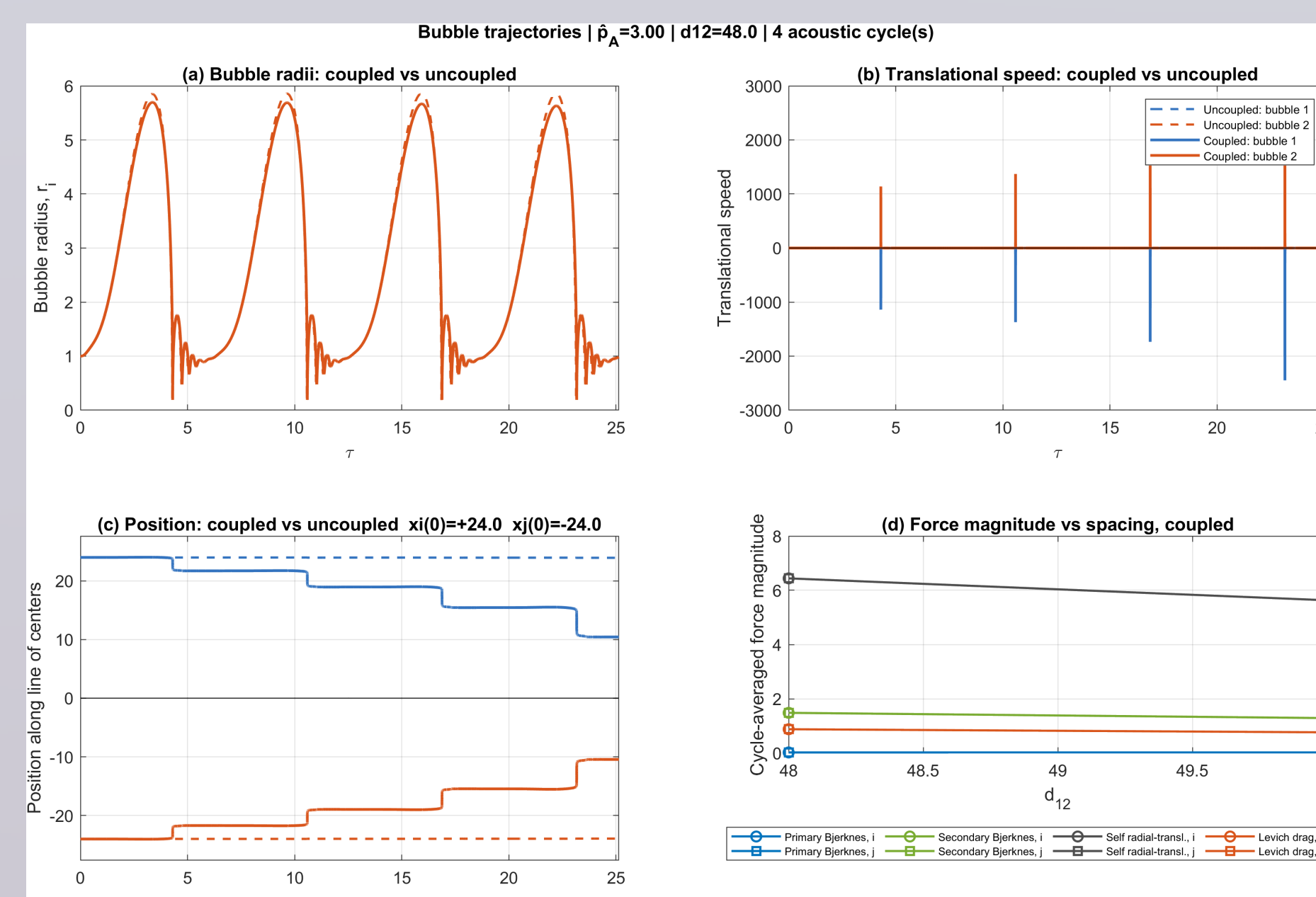


Figure 2: Radius, speed, and position vs. time; force vs. spacing.

Key Observations

- 2-bubble, $\hat{p}_A = 2$: secondary Bjerknes dominates; bubbles attract.
- 2-bubble, $\hat{p}_A = 3$: self radial-translational dominates; migration $\sim 100\times$ larger.

Key Observations (cont.)

- 3-bubble, $\hat{p}_A = 2$: secondary Bjerknes decreases with separation while Levich drag remains dominant; weak inward migration.
- 3-bubble, $\hat{p}_A = 3$: self radial-translational dominates; strong inward migration.

Three equal-size bubbles, $\hat{p}_A = 2$

$r_{0,1} = r_{0,2} = r_{0,3} = 1$, 4 acoustic cycles, coupled vs. uncoupled.

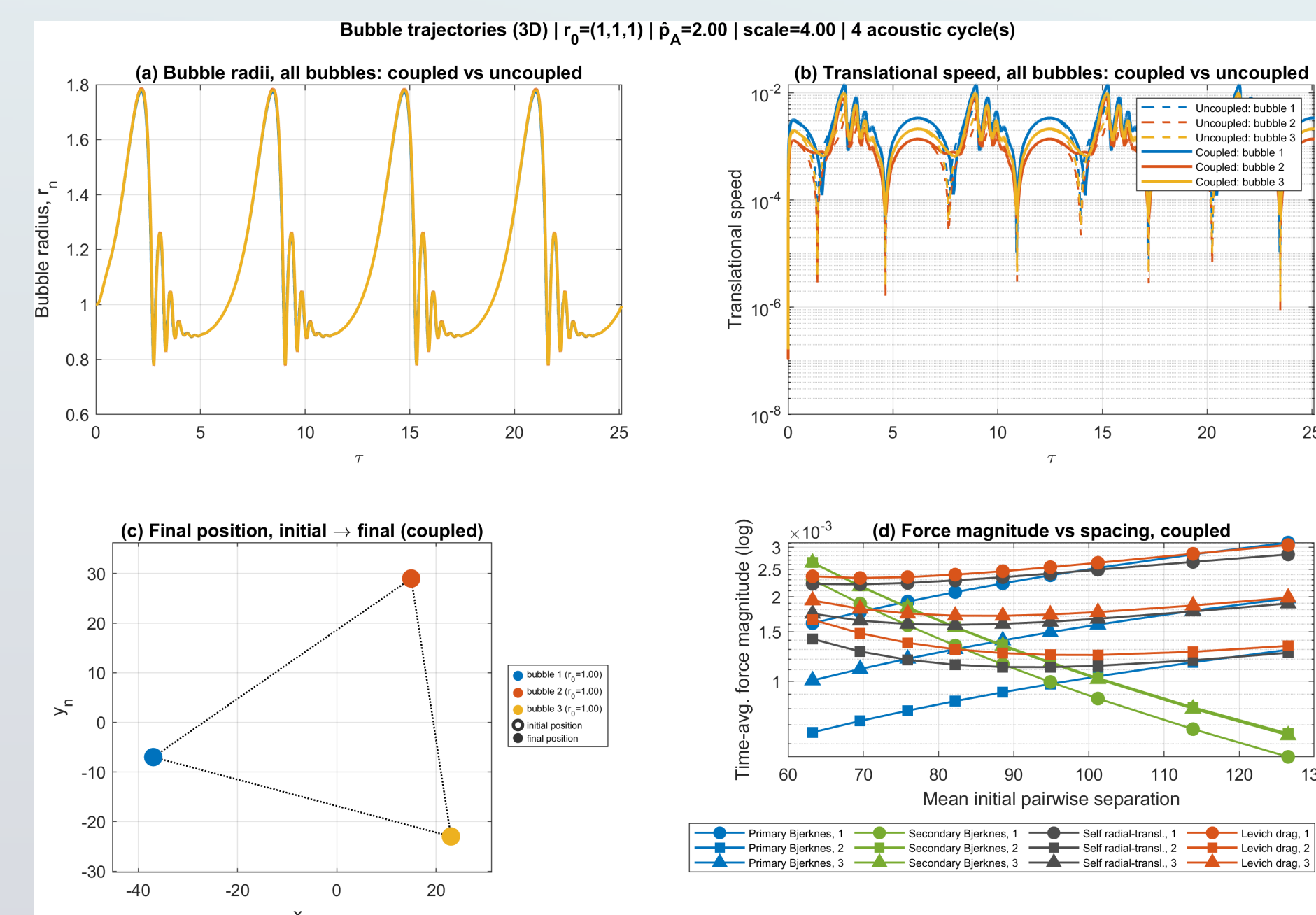


Figure 3: Radius and speed vs. time; final position and force vs. separation.

Three equal-size bubbles, $\hat{p}_A = 3$

$r_{0,1} = r_{0,2} = r_{0,3} = 1$, 4 acoustic cycles, coupled vs. uncoupled.

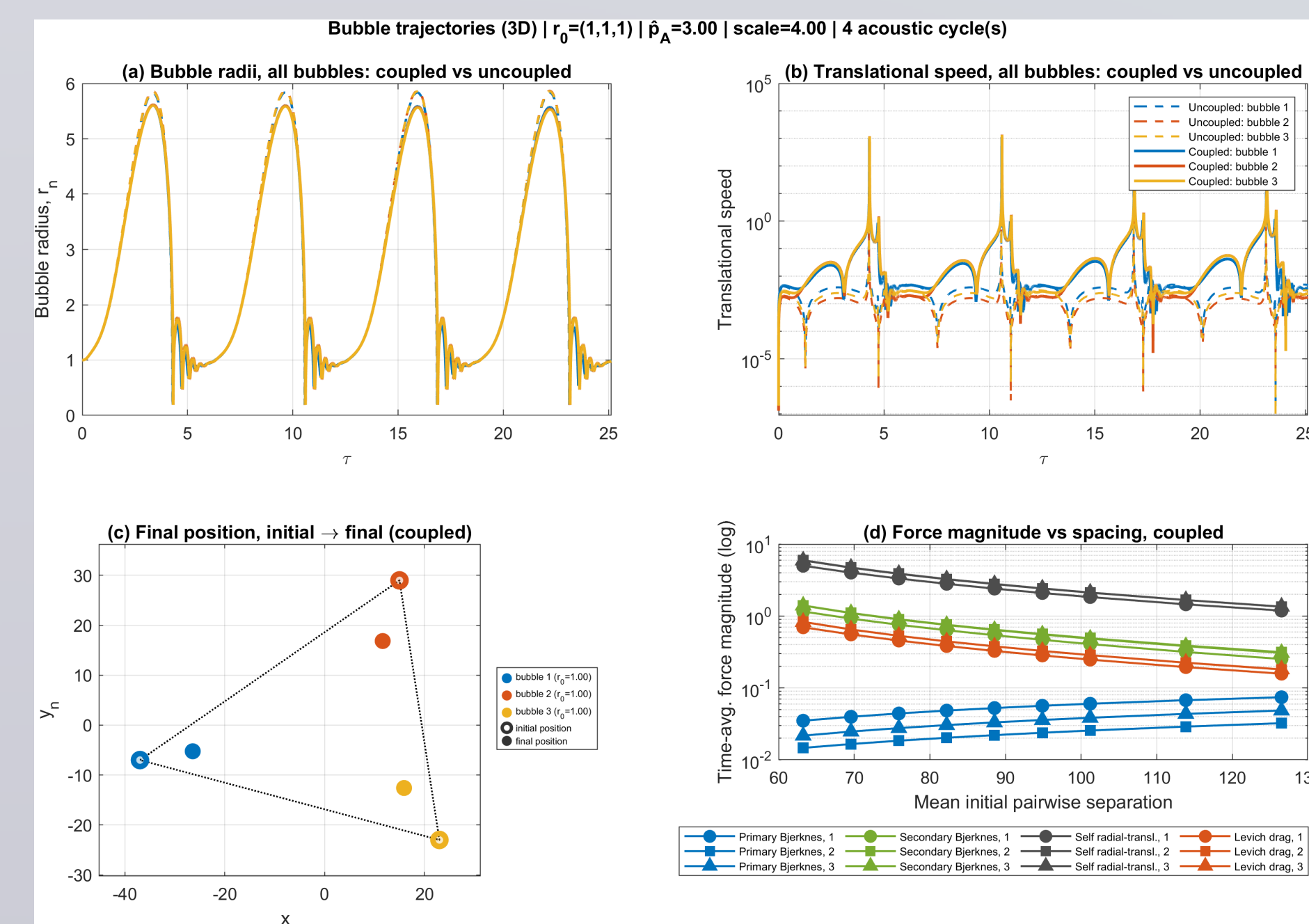


Figure 4: Radius and speed vs. time; final position and force vs. separation.

Conclusions & Future Work

Forcing amplitude determines which coupling term dominates. At $\hat{p}_A = 3$, the self radial-translational term drives migration orders of magnitude larger than at $\hat{p}_A = 2$. Next we will investigate the effects of a broader range of forcing amplitudes, unequal bubble sizes, larger clouds, and mass and shape stability.

References

- [1] Doinikov, J. Acoust. Soc. Am. (2004)
- [2] Brenner, Hilgenfeldt, Lohse, RMP (2002)
- [3] Meegoda *et al.*, IJERPH (2022)
- [4] Qin *et al.*, Ultrason. Sonochem. (2024)